Chapter 5:

And all the while, the theorists.

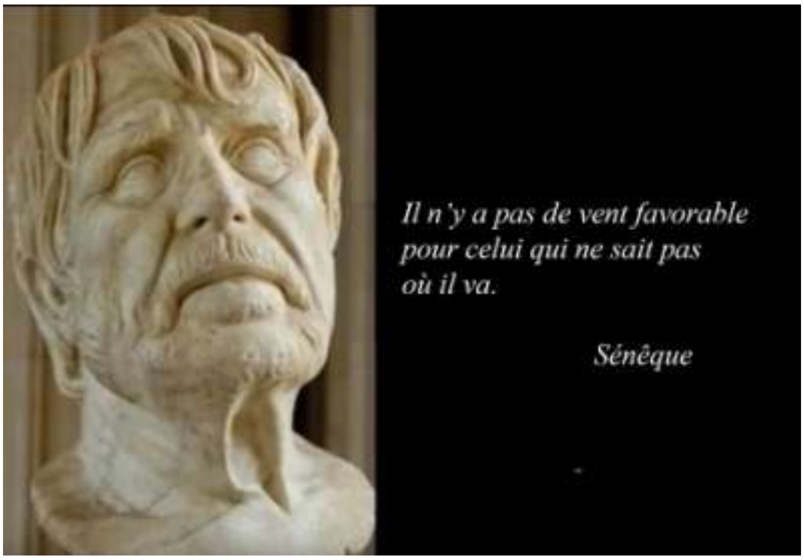
We have briefly mentioned the failure of the extension of the particle bestiary through supersymmetry. One candidate particle had particularly caught the attention of theorists, namely the supersymmetric companion of the neutron, the neutralino. This one has been considered as the best candidate as a component of dark matter. Experiments were tried, in different countries, to try to capture it. It was assumed that this neutralino would interact only very weakly with ordinary matter. The desired signal must therefore be extremely weak. It was therefore essential to protect oneself from the background noise linked to the bombardment of the Earth by cosmic rays. For this purpose, laboratories have been installed in road tunnels, under thousands of meters of rock, or at the bottom of mines. The experiment in which the scientists had the most hope was to record the signal from the interaction of this neutralino with a mass of liquid Xenon. So we tried the experiment with one kilo of xenon, then ten, then a hundred, then a ton. Today there are detectors with 4 and 6 tons of xenon. But no results.

We are then confronted, not with a scientific problem, but with a psycho-sociological problem. How can we admit that this path is a dead end, when we do not allow ourselves to imagine any other?

The string theory, born in the sixties, represents another example of maze in which thousands of researchers got lost. It is necessary to be situated at this time. Particle theorists realize that there is something wrong with their approach. This seems to have reached a plateau. Under these conditions it was not illogical to look for a totally different approach. String theory presented itself as a totally new way, a different approach to problems. It wanted to be able to provide solutions to all problems, by acting as a "theory of everything"[#1#] . Its major flaw was its difficulty in connecting to reality. It means to explain a phenomenon or an observation, to predict something, to provide a model of an object, a modeling of a process.

Basically there was an extension of the dimensional context, beyond four, to ten and even eleven dimensions. One is tempted to quote the phrase of the Latin philosopher Seneca

[1] In English: TOE "Theory of Everything". In fact TON : "Theory of Nothing".



Indeed, if we venture into the arcane history of science, we can see that behind the most sophisticated formalisms there are always ideas, a program, a plan. Mathematics is there to help the theorist. In physics, they are not an end in themselves. But they are useful, necessary. The mathematician Jean-Marie Souriau said:

- Mathematics is like a shoe. You can walk without shoes. But with them, you can go further and faster.

For those who have a mathematical level corresponding to that of special mathematics, I will end this book with pages that only they can read and appreciate. However, I will describe the steps without resorting to this mathematical translation, in a form that remains accessible to non-mathematicians. In what follows we will see the dead ends to which people have committed themselves who, precisely, have undertaken to play with formalisms while completely losing sight of the guiding ideas.

We must always keep in mind that these mathematics are only tools. The underlying ideas, always very simple, are always much more important than the formalisms with which they are expressed. In any case, just because the mathematical writing is complicated does not mean that this approach is the right one:

From this point of view, we can say that the fifty years of buryrocinesis[#2#] represented by the string theory are the example of a formalism without any real guiding idea. The operation leads to a fantastic aberration, to a combinatorial system that can generate 10^{500} possible theories.

Forgetting that 10^{500} times an absence of idea gives an absence of result.

When I was a schoolboy we had an art teacher who told us:

- When you make a sketch, combine a multitude of strokes. Then the right one will necessarily be in it.

The people of the strings are like draughtsmen who have drawn so many lines on a sheet of paper that it takes on a uniform black hue, and who say: "the most beautiful drawings in the world are automatically on this sheet of paper, because it contains them all.

It is not known what will become of this pseudo theory. Its champions continue to talk about it. There are pages on Wikipedia where the different variants are listed, often with strange names, such as ekpyrotic (from the Greek ekpyros: "blazing"). Creating a new word, preferably grandiloquent, is generally a way used by many to give the illusion of progress.

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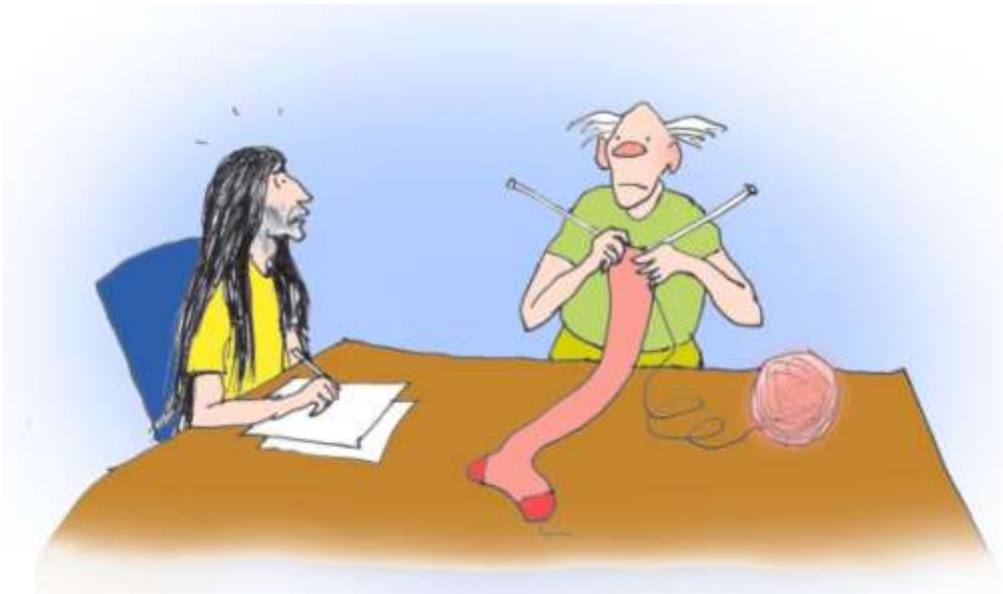
[2] From the Greek butyros, butter, and kinesis, movement.



Super strings could be reused to make super socks.

We have seen that in the nineteenth century the equations allowing to have a grip on the phenomena of Nature had appeared. In these equations we find a number of quantities, presented as constants of physics.

c : the speed of light

G : the observation of the gravitation.

m : "the masses of the different particles".

e : the elementary electric charge.

ε_o : the dielectric constant of vacuum.

μ_o : the magnetic permeability of the vacuum.

h : Planck's constant.

α : the fine structure constant.

R_b : the Bohr radius

etc. .

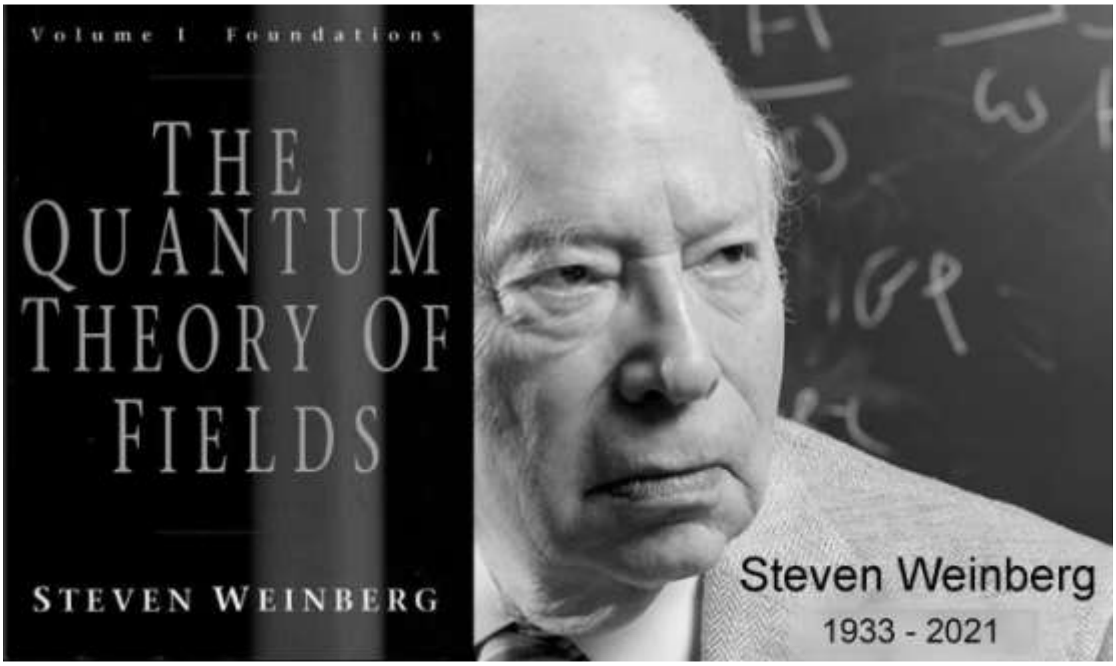
All are not independent. For example the speed of light, the dielectric constant and the magnetic permeability of vacuum are related by the relation :

$$c = \frac{1}{\sqrt{\mu_o \varepsilon_o}}$$

Scientists therefore wondered whether these quantities deserved their name of absolute, invariable constants. They have therefore considered that these constants may have varied over the course of cosmic evolution. They studied their possible variations, taking them one after the other. All sorts of contradictions then appeared. In one case the atoms could no longer be formed, etc. It was then considered to couple variations of different constants. But contradictions remained.

Apart from that, decades have passed without any progress in the search for the nature of dark matter and dark energy. But then, what had happened to bring our theoretical disciplines to such a dead end?

Not being a specialist in quantum mechanics, I will content myself with this remark on the nature of space and time inversion operators.



The Quantum Field Theory Bible

In the book this is located in the section entitled: 2.6 Inversions of space and time.

And more precisely at the step of page 75 and in page 76. The space inversion operator is called P (P for P-symmetry, , "parity") . This is mirror symmetry. La lettre T is the time inversion operator. Weinberg states that these two operators can be:

- Linear and unitary

ou :

- Anti linear and anti unitary.

In order to avoid the emergence of negative energy states, considered a priori as impossible, it indicates that one is obliged[#3#] to opt for the choices :

- P Linear and unitary

- T Anti linear and anti unitary.

To this one will object what follows from the observation of the acceleration of the cosmic expansion. This is due to a negative (dark) energy, a priori sum of negative energy states.

I will come back later on to the reasons that lead me to think that the addition of negative masses is the key to the quantization of gravitation.

End of this short digression in a field that is not mine.

So I come back to cosmology and theoretical astrophysics.

As I said above, in 1915 Albert Einstein published the field equation which forms the basis of the general relativity model. A few months later, the mathematician Karl Schwarzschild published two articles in quick succession, describing the geometry outside and inside a sphere filled with an incompressible material. Unfortunately, shortly afterwards, he died of an infection contracted on the Russian front, which he had joined as a volunteer.

I will try to present what it is about without using mathematical tools which would have the effect of arguing it many readers of this book. It is not necessary to understand all the ins and outs of these solutions of Einstein's equation to perceive what happened at that time and during the years that followed, until the outbreak of the Second World War.

These solutions are expressed in the form of algebraic expressions called metrics. Here is the one in Schwarzschild's writings that describes the geometry outside the fluid ball:

[3] « There are no states of negative energy energy less that that of the vacuum) so we are forced to choose …" , translation: so we are obliged to choose ... and a little further on: "... disastrous conclusion".

Außerhalb der Kugel bleibt die Form des Linienelements dieselbe, wie beim Massenpunkt:

$$ds^2 = \left(1 - \frac{\alpha}{R}\right) dt^2 - \frac{dR^2}{1 - \alpha/R} - R^2 (d\vartheta^2 + \sin^2 \vartheta \, d\phi^2) \qquad (36)$$

wobei:

$$R^3 = r^3 + \rho$$

ist. Nur wird ρ nach (33) bestimmt, während für den Massenpunkt $\rho = \alpha^3$ war.

§ 7. An die im vorigen Paragraphen enthaltene vollständige Lösung unseres Problems knüpfen sich folgende Bemerkungen.

The geometric Schwarzchild solution describing the exterior of the ball

It is a function where we distinguish variables and a parameter, represented by the letter α. . It is a positive quantity, a length to which we will later give the name of "Schwarzschild radius".

We have four variables, which is normal since this expression refers to a space with four dimensions. The first is designated by the letter t and is the time coordinate. Then we have two angles[#4#] : #&

$$\vartheta \text{ et } \phi$$

The last variable is designated by the letter R, and will be the source of an error on which hundreds of doctoral theses and thousands of articles will be based!

Before we focus on this letter r Let's start by getting rid of a Greek letter, which to read is simply equal to . So, let's simplify. It comes:

$$ds^2 = \left(1 - \frac{\alpha}{R}\right) dt^2 - \frac{dR^2}{1 - \alpha/R} - R^2 (d\vartheta^2 + \sin^2 \vartheta \, d\phi^2)$$

$$R^3 = r^3 + \alpha^3$$

What is this letter r minuscule ? Its meaning is immediately given to us by Schwarzschild. Initially he imagines expressing his solution using a time coordinate t , and three three-dimensional "Cartesian" space coordinates :

x , y , z

But, right away, he poses:

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[4] The student will recognise the index of a representation of this "object" in polar coordinates.

Schwarzschild: Über das Gravitationsfeld eines Massenpunktes 191

$$ds^2 = F dt^2 - G(dx^2 + dy^2 + dz^2) - H(x dx + y dy + z dz)^2$$

wobei F, G, H Funktionen $r = \sqrt{x^2 + y^2 + z^2}$

Die Forderung (4) verlangt: ... $= 1$, $H = 0$.

Wenn man zu Polarkoordinaten gemäß ... $r \sin \vartheta \cos \phi$, $y = r \sin \vartheta \sin \phi$, $z = r \cos \vartheta$ übergeht, lautet dasselbe Linienelement:

Thus, the minimum value of r est zéro. Under these conditions the minimum value of r est :

$$R = \alpha$$

Obviously, the "radial variable" is r et non *R*. In his paper, Schwarzschild calls it «Hilfsgröβe » i.e. of "intermediate size".

→And then someone will make the mistake of confusing r and r, to treat r as a radial variable, whereas it cannot be less than αthat is, the Schwarzschild radius, which we will call R_g.

We will identify the author of this (transcription) error later. To this will be added the inversion of the signs in the expression, and we will also look for who is at the origin of this choice. Still, a century later, in 2015, the academician Thibault Damour "Mr. Cosmology in France" gave a conference on the occasion of the centenary of the creation of general relativity.

Thibauld Damour, académicien

This takes place in the large lecture hall of the Institut des Hautes Études de Bures sur Yvette, near Paris. This document can be accessed by anyone at the following Internet address

https://youtu.be/SqGlFifHBfo

At 23 minutes and 6 seconds he projects the following image:

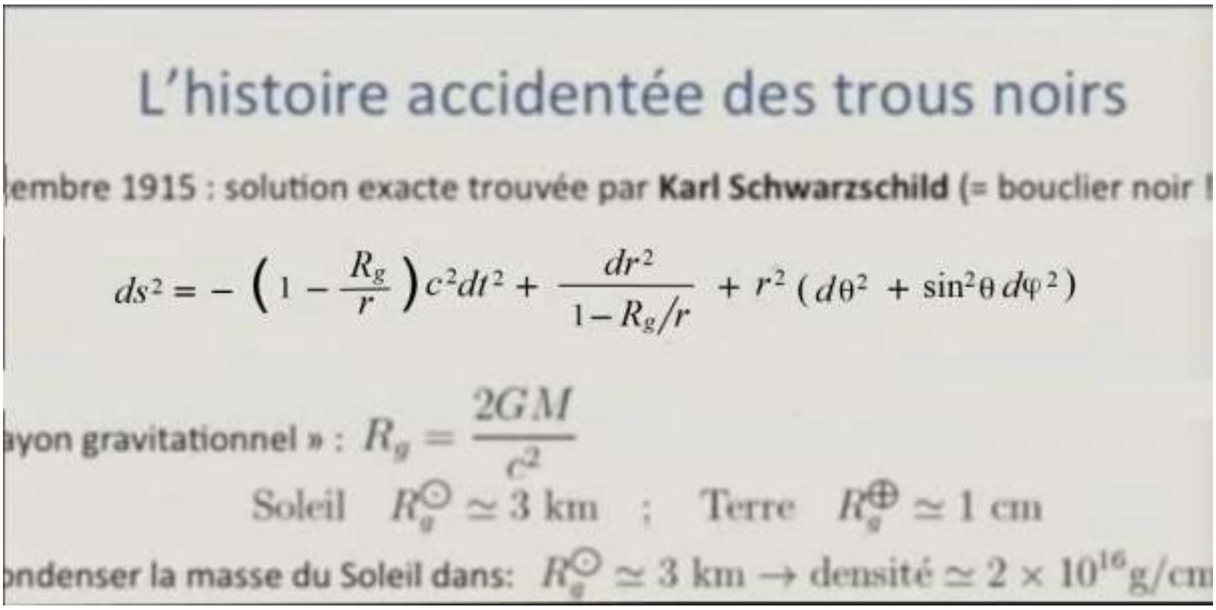
On the right the photograph of Karl Schwarzschild. Just below his famous solution. Let's enlarge :

One can immediately recognize, below, the expression presented above, as it emerges from the original article in German. Just above, the definition of this «intermediate size r » from which the constraint instantly emerges:

$$R \geqslant \alpha$$

The following image, shown by Thibault Damour, at 25 minutes and 2 seconds:

Let's enlarge the right part of the image:



This represents the "modern" interpretation of the Schwarzschild solution, describing the geometry outside the mass. The letter was replaced by the Schwarzschild ray.

But the letter *r* has been substituted for the letter *R*
and the constraint has simply disappeared!

Incidentally, the signs have been reversed. We will come back later on to the origin of this change of signs.

The consequence is that from then on our modern theorists will consider for half a century to study the structure of this "geometrical object" when r is less than α, smaller than the Schwarzschild radius R_g, i.e. "inside this sphere" ... which does not exist!

How is it that the speaker does not realize for a second the contradiction that arises from the display of these two images?

I think the answer lies in the mindset of modern theorists and the idea of scientific progress. For 50 years, they have been building their speeches on the speeches of their predecessors, having totally lost all critical thinking, all hindsight. If you were to ask them:

- But, why did you replace this letter R, simple intermediate size, as defined by Karl Schwarzschild in 1916, by the letter r, which you then treat as a radial variable, able to take all positive values, up to zero?

You might get the following answer:

- Quand r becomes smaller than the Schwarzschild radius, we are inside the Schwarzschild sphere R_g. At that moment this variable r no longer designates the radius, but the time, while t becomes a space variable. These two variables exchange their respective roles. It took a long time to understand this. It was violent[#5#] (...).

As a comment, I think that this sentence of the French mathematician Jean-Marie Souriau, whose teaching I collected for more than twenty years, seems appropriate:

[5] This is the term used by Damour.

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-
-
- - At this stage, it is necessary to look for the source of this drift.
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-
- I said above that Hilbert had published, on November 20, 1915, an article entitled "The Foundations of Physics". He then discovered the first article published by Schwarzschild, in January 1916, which describes the geometry outside a mass but his attention stops there. He does not pay attention to the second one, the one from February, which describes the geometry inside a ball filled with a material of constant density. He decided to integrate this first result in a second version that he published on December 23, 1916, just before Christmas. These two documents can be downloaded in their French translation from the following addresses
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- http://www.jp-petit.org/papers/cosmo/1915-Hilbert-fr.pdf
-
- http://www.jp-petit.org/papers/cosmo/1916-Hilbert-fr.pdf
-
- - The first mistake is to consider only one of these geometric solutions. Indeed, these solutions do not make sense if considered in isolation.

We had said above that the emergence of a new physics was presented as a complete critique of common sense. This is totally true in quantum mechanics, which abounds in paradoxes. I describe them in Janus videos.

The Young's slit paradox in Janus 5: https://youtu.be/aLPpCQRfwRg

The Einstein-Podowlski-Rosen paradox in Janus 6: https://youtu.be/0Uw0LifxA_A

This quantum mechanics had its leader, the Dane Niels Bohr. Although Einstein was always very aware of the advances of this new way of dealing with reality, he remained reticent about the probability aspect underlying the whole edifice. We know the exchange between Einstein and Bohr:

Einstein:

- I refuse to believe that God plays dice!

To which Bohr had replied:

- But who are you to tell God what to do!

Niels Bohr and Albert Einstein around 1930

Until his death Einstein tried to take this quantum mechanics in defect. He thought he found the loophole with the EPR paradox, see the video Janus 6. In this experiment an atom emits two photons, in diametrically opposite directions. We then act on one of them by rotating its "polarization plane"[6]. But the other photon is immediately "warned" and also

[6] Thanks to the effect discovered by the Englishman Faraday, see the video Janus 6.

turns its own polarization plane accordingly[7]. Common sense would suggest that these two photons manage to communicate and that this information would then travel at twice the speed of light.

The calculations of quantum mechanics predicted that it would be so. Unfortunately, both Bohr and Einstein were dead when the Frenchman Alain Aspect brought the experimental confirmation in the early eighties[8].

Thus, the absurd, that which violated common sense, appeared closer to physical reality.

Does this mean that this common sense can no longer be useful to us?

You have to have it both ways.

- At the same time be ready to admit that mathematical elucubrations can suggest aspects of reality that are completely opposed to this common sense. Until, of course, Nature expresses itself by confirming the predictions of this model (which has not been the case)

- But also to be always ready to reconsider some of these mathematical elucubrations under the optical system of this common sense, when they are too slow to produce a confirmation with experience and observation.

As far as general relativity is concerned, we will see that this second attitude prevails. The approach of the problems by using images and analogy, makes it possible to approach the realities, i.e. in cosmology, the observations. Intuition remains a useful tool.

What is modeling aspects of the cosmos using general relativity? Let's focus on stationary, time-independent solutions.

These are based entirely on Einstein's equation. Its second member takes into account the content of the universe, at any point.

$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} = \chi\, T_{\mu\nu}$$

If it is non-zero, then the geometric solution that will emerge from this equation will describe the interior of a mass.

If it is zero, the equation becomes :

$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} = 0$$

[7] So that the two planes remain orthogonal, as predicted by quantum mechanics.
[8] This earned him the Nobel Prize in 2022, forty years later.

The solution that will emerge from this equation will refer to a totally empty portion of the universe.

In 1916, with this suite of two papers, Schwarzschild builds in the first (January) the geometry outside the mass, and in the second (February) the geometry inside the mass.

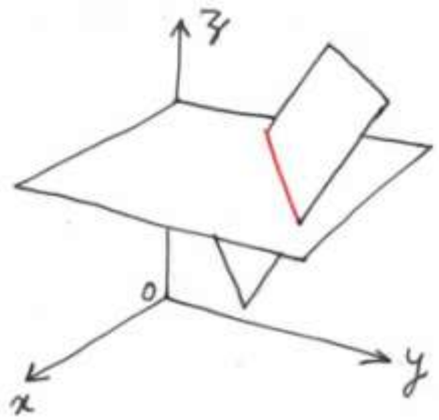
In 1916 a young mathematician (he was thirty years old) brought his own reading of Schwarzschild's work. His name is Ludwig Flamm. His work, and even his name, remain totally unknown to cosmology specialists, for a simple reason: his article was only translated into English in 2012!

It is difficult to get a copy, not only in English, but even in its original form, in German language, if not by paying. For this reason we have translated it into French and you can download it at this address:

http://www.jp-petit.org/papers/cosmo/1916-Flamm-fr.pdf

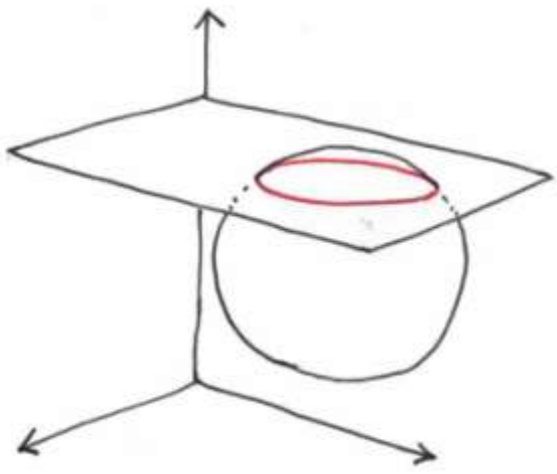
Since we limit ourselves to time-independent solutions (invariant by time translation) we are reduced to the understanding of a three-dimensional hypersurface (at constant t). Flamm approaches the question by making cuts of it, and it is remarkable.

We will generalize this concept of cut (by assigning to one of the coordinates a constant value). Consider a plane "immersed" in a three-dimensional space. This plane is a 2D surface without curvature. If we cut this plane at z = constant, we will obtain a one dimensional object (2 - 1), a straight line, a "Euclidean" object.



Plane couple of a plane

On the other hand, if we perform this cut at z = constant of a 2D object with curvature, for example a sphere, the result is still a one-dimensional object, but it is no longer a line.



Flat section of a sphere S2

Performing a "cut by plane" reduces the number of dimensions of an object by one. If we consider a section of a three dimensional hypersurface, this section will be a two dimensional object. If this hypersurface has a curvature, then this section will differ from a plane.

Let us imagine a three-dimensional hypersurface with constant curvature. It is a "hypersphere", which obviously cannot be associated with a mental image[9]. If we make "cuts" by "plans r = Constant, these cuts will be spheres S2[10] .

The section of a sphere S3 by a plane.

The following is a translation of the expression of Schwarzschild's inner and outer solutions as presented by Flamm in 1916:

[9] Its metric would be $ds^2 = dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$

[10] In the expression of the metric it is sufficient to make dr = 0 and one falls on the metric of a sphere of radius r.

Flamm 1916

2

$$d s^2 = \left(\frac{3\cos\chi_a - \cos\chi}{2}\right)^2 d t^2 \; - \frac{3}{\kappa\rho_0}\left(d\chi^2 + \sin^2\chi\, d\vartheta^2 + \sin^2\chi\,\sin^2\vartheta\, d\varphi^2\right)$$

où les coordonnées ont déjà été choisies d'une manière particulière et appropriée. Voici une coordonnée qui augmente radialement à partir du centre de la boule de fluide, atteignant la valeur χ_a à la surface limite, et ϑ et φ sont les coordonnées sphériques habituelles. La constante ρ_o représente la densité de la boule de fluide et κ représente la constante gravitationnelle de la théorie d'Einstein, qui a pour valeur

$$\kappa = \frac{8\pi k^2}{c^2},$$

Où κ est la constante gravitationnelle habituelle

$$k^2 = 6.68 \cdot 10^{-8}\mathrm{cm}^3\mathrm{g}^{-1}\mathrm{sec}^{-2}$$

Et c la vitesse de la lumière.

Ludwig Flamm 1916 :

§2 . Nous pouvons maintenant, de manière analogue, examiner le premier cas traité par Schwarzschild, le champ gravitationnel d'une masse ponctuelle. L'élément de ligne, dans sa forme la plus simple, a la même structure que ci-dessus et se lit comme suit

$$d s^2 = \left(1 - \frac{\alpha}{R}\right) d t^2 - \frac{d R^2}{1 - \frac{\alpha}{R}} - R^2\left(d\vartheta^2 + \sin^2\vartheta\, d\varphi^2\right)$$

Une quantité est entièrement nouvelle dans cette expression, la constante α^1, qui a la valeur suivante

$$\alpha = \frac{2k^2 M_0}{c^2},$$

où *Mo* représente la masse centrale, telle qu'elle serait obtenue à partir de mesures astronomiques. L'élément de ligne sera à nouveau décomposé selon la formule (1).

We will see later how to express the two solutions with the same set of coordinates.

Le terme k^2 is simply the way the gravitational constant G was represented at that time.

The description of the geometry given by Schwarzschild passes through the connection between two hypersurfaces, along a sphere (in this case the surface of this sphere filled with an incompressible material of constant density). The first hypersurface is a three-dimensional hypersphere, defined by the Schwarzschild inner solution[11] of constant curvature. Its plane section by a plane = constant is therefore a sphere S2.

The second one is defined by the Schwarzschild outer metric solution[12]. Since these geometries are independent of time, we can start by considering sections at constant t. We then obtain a three-dimensional hypersurface:

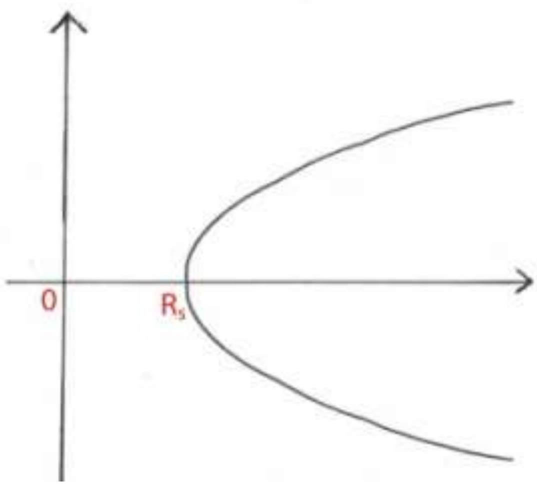
$$r, \theta, \varphi$$

Flamm will then consider a section at θ constant. It will then obtain a two-dimensional geometric object:

$$r, \varphi$$

Earlier, when we made a cut of our three-dimensional hypersphere the resulting section (a circle) had inherited part of the symmetry of the 3D object. The 3D Schwarzschild hypersurface has spherical symmetry. The section that we realize inherits a part of this symmetry in the sense that this 2D surface has a symmetry of revolution.

Flamm says to himself "let us operate again a section of this 2D surface by planes with φ constant ». And it then obtains a one-dimensional object, a curve which is a "lying parabola":



Reclining parabola

It is nothing else than the meridian of this surface of revolution which represents the section by a plane of the external hypersurface solution, given by Schwarzschild whose equation is (go check in the article) :

[11] His second paper, of February 1916

[12] The first paper, of January 1916

Flam 1916

$$\cos^2 \chi = 1 - \frac{\alpha}{R}$$

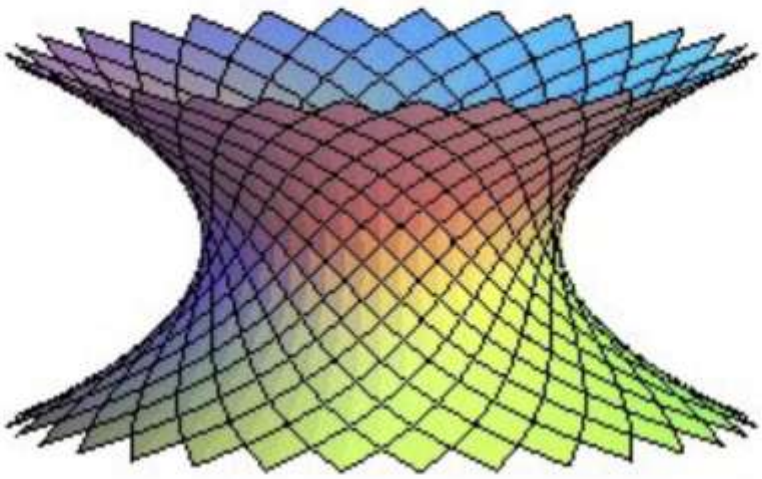
En désignant par z la coordonnée selon la direction de l'axe de rotation, l'équation de la courbe méridienne suit alors :

$$\frac{dz}{dR} = \operatorname{tg} \chi = \sqrt{\frac{\alpha}{R - \alpha}}$$

ou :

$$z^2 = 4\alpha(R - \alpha).$$

Ceci correspond à une parabole



In Wikipedia the "Flamm surface

This surface[13] has a "throat circle", of minimum perimeter 2 Rs, where Rs is the Schwarzschild radius. It connects two planes. Indeed the curvature of the surface tends to zero at infinity[14].

Considering now that this object corresponds to a constant cut of the hypersurface corresponding to the Schwarzschild outer solution, and although this is somewhat difficult to

[13] This is a result that we had regained in 2015. See: http://www.jp-petit.org/papers/cosmo/2015-ModPhysLettB.pdf

[14] The Flamm surface has a negative curvature. But the solution geodesics of the Einstein equation are 4D geodesics. The masses do not follow the 2D geodesics of the Flamm surface.

picture for the average person, the throat circle of this Flamm surface, declined for all values of , will envelop a sphere.

of area $4\pi R_s^2$

Which constitutes the sphere of throat of the hypersurface which connects between them two 3D euclidean spaces.

What is important, crucial to understand is that, just as the lying parable "does not exist" for values :

$$0 < r < Rs$$

Simply because the value of z from the meridian equation, see above, is no longer real.

Similarly the surface of Flamm "does not exist" for the same values, corresponds to the interior of a cylinder of perimeter $2\ \pi R_s$

Similarly the hypersurface of the Schwarzschild outer solution "does not exist" inside a sphere of area $4\pi R_s^2$

You will notice that I purposely did not use the word "center" and "radius". Because this point does not belong to this geometric object.

We have said above that the 3D hypersurface corresponding to the outer solution found by Schwarzschild has the topology of an S3 sphere. This one can be laminated by a series of cuts to θ constant. These cuts will be objects of revolution, in this case spheres. As for the previous object, Flamm operates a new section by planes with φ constant and it obtains an arc of circle. By adapting this second meridian, he thus obtains the complete meridian of the section at θ constant of the hypersurface solution.

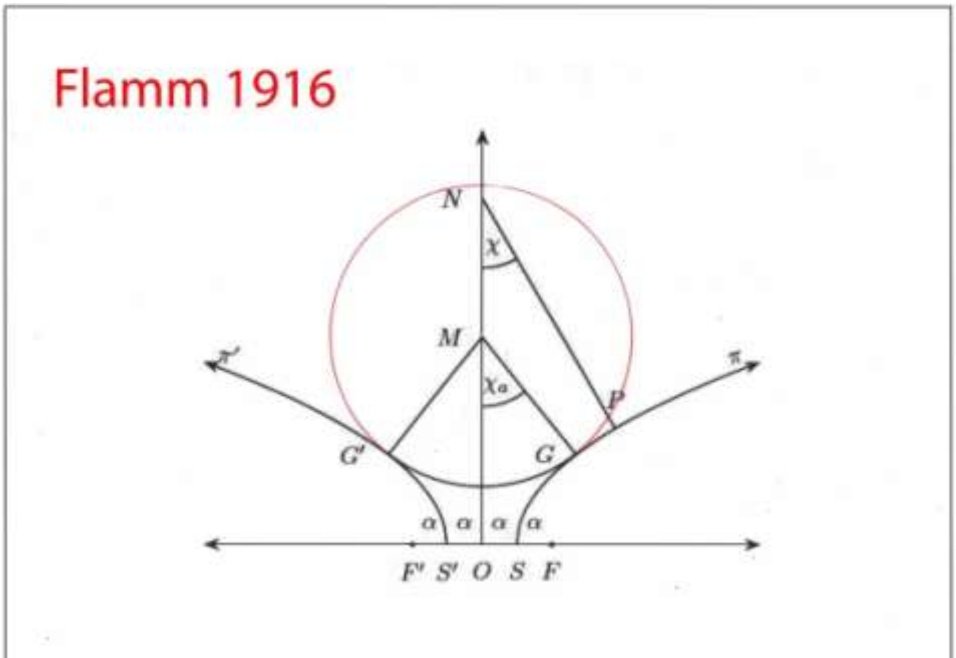


Fig. 3

All this translates, in 1916, into a perfect mastery of the geometry of these geometric objects that are the three-dimensional Riemannian hypersurfaces, which is totally lacking in today's theorists.

In order to help the reader, we will start by transforming the inner Schwarzschild solution so that it is expressed with the same variables as the outer solution.

The change of variable, we have seen above. Let us rewrite the inner solution by making the constant of gravitation appear and by making the speed of light reappear, taken equal to unity in Flamm's paper, as in those of Schwarzschild.

$$ds^2 = \left(\frac{3\cos\chi_a - \cos\chi}{2}\right)^2 c^2 dt^2 - \frac{3\,c^2}{8\pi G\rho_0}\left(d\chi^2 + \sin^2\chi\, d\vartheta^2 + \sin^2\chi\,\sin^2\vartheta\, d\varphi^2\right)$$

$$\hat{R}^2 = \frac{3\,c^2}{8\pi G\rho_0}$$

All calculations done, it comes[15] :

[15] Expression found in chapter 5 of Adler, Schffer and Bazin's "Introduction to General Reltivity", Mc Graw Hilled. Download at http://www.jp-petit.org/books/asb.pdf

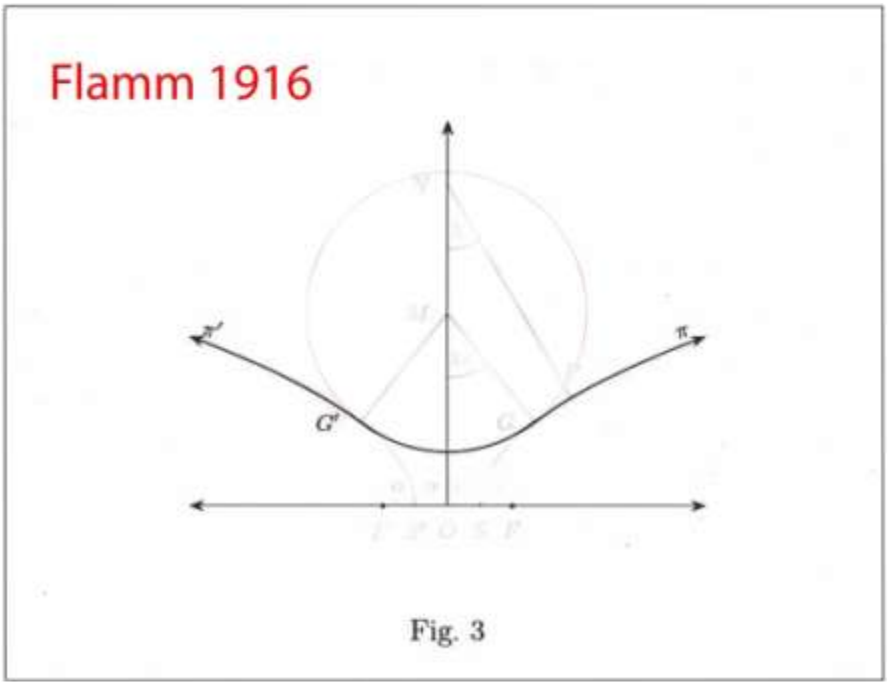
$$ds^2 = \left[\frac{3}{2}\sqrt{1 - \frac{R_a^2}{\hat{R}^2}} - \frac{1}{2}\sqrt{1 - \frac{R^2}{\hat{R}^2}} \right]^2 c^2 dt^2 - \frac{dR^2}{1 - \frac{R^2}{\hat{R}^2}} - R^2 \left(d\vartheta^2 + \sin^2\vartheta \, d\varphi^2 \right)$$

The solution of the problem treated by Schwarzschild consists in joining two portions of solution, two parts of hypersurfaces.

For the inner solution, only the portion corresponding to r <Ra
(corresponding to the interior of the star).

We will complete with the external solution, taken when R>Ra (external to the star).

The meridian will correspond to a part of the meridian of the Flamm surface, connected to an arc of circle:



The meridian of the hypersurface-solution

In the outer geometric solution we have seen a constraint appear, that of considering it only for the values R> , element whose value is indicated in a box, above, otherwise the element of length ds is no longer real, we are then outside the hypersurface.

We know that for the Sun this length is 3 kilometers.

But a second constraint for the inner solution appears. If we have a denominator that becomes zero. So we will only use the solution part if :

$$\boxed{R < \widehat{R} = \sqrt{\frac{3\,c^2}{8\pi G\rho_0}}}$$

Let's apply this to the Sun. What is this length worth? Answer: 580 times the radius of the star.

But what is the geometric significance of this length? It is easy to verify. It is simply the radius of the circle from which we will take an arc to complete the meridian.

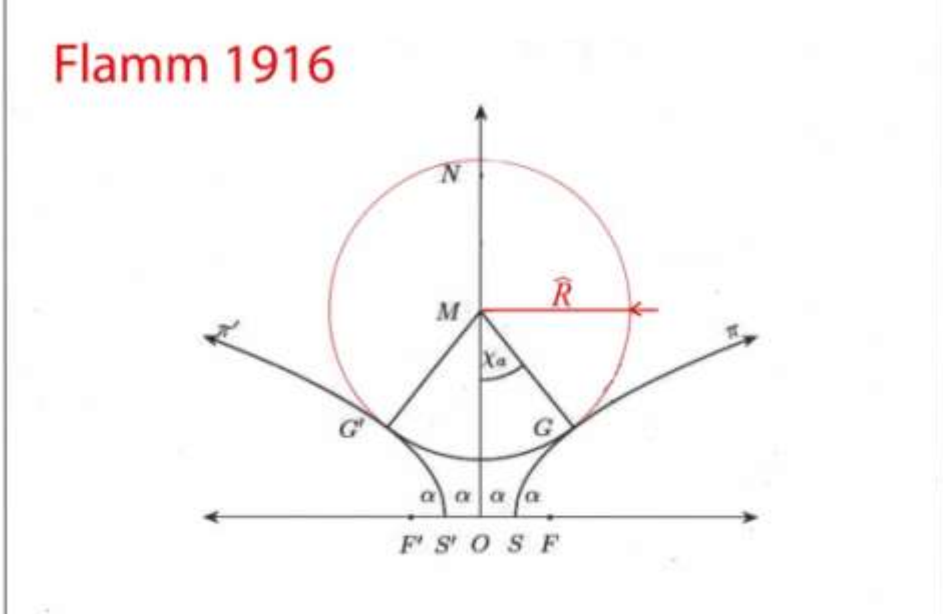


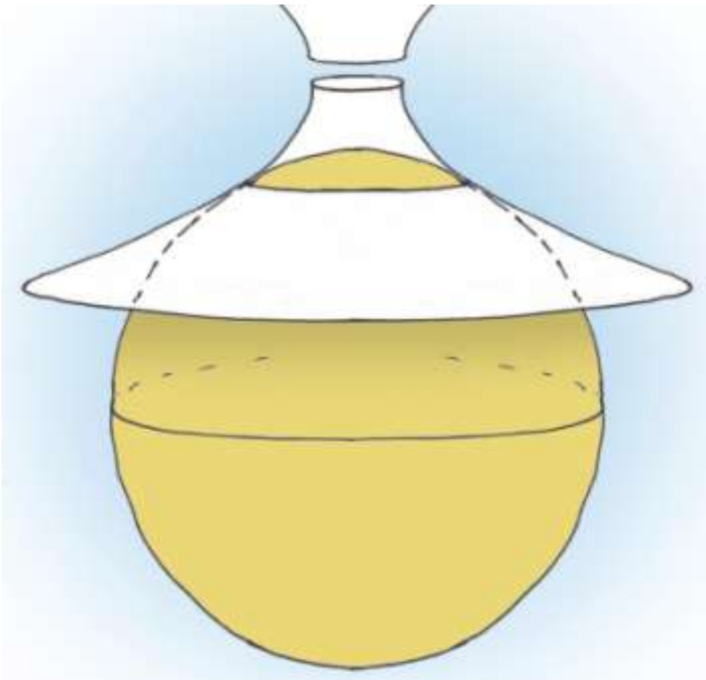
Fig. 3

The reader unfamiliar with algebra may feel a little lost. Let's forget all these ugly formulas and concentrate on the drawings, on the geometry.

By the way, we are going to turn the figures upside down, the notion of curvature being more familiar when we talk about bumps and not hollows. The following image is not a didactic image. It really is the solution. This is the 2D section of the 3D hypersurface solution.

How is it made?

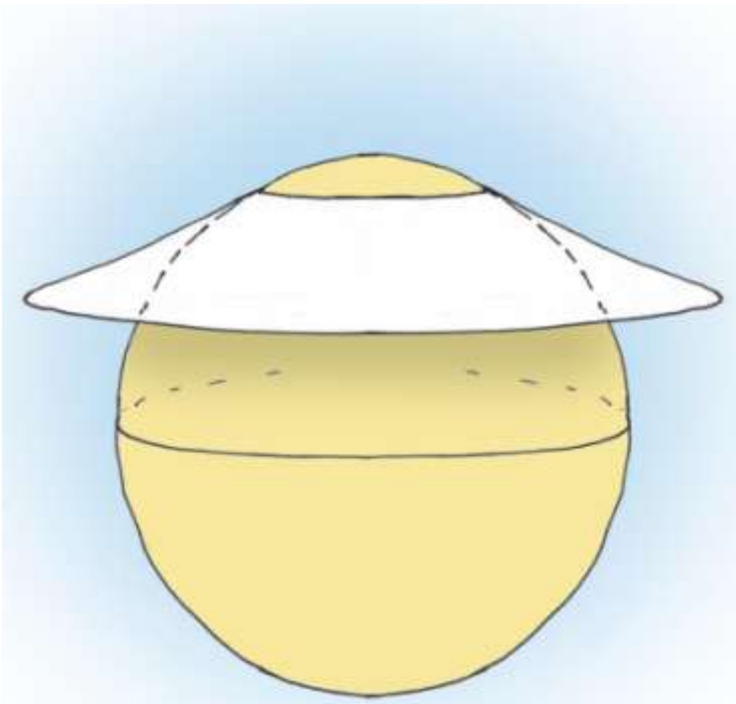
You take half of a Flamm surface (obtained by rotating a parabola around an axis). You then plate a sphere, like this:



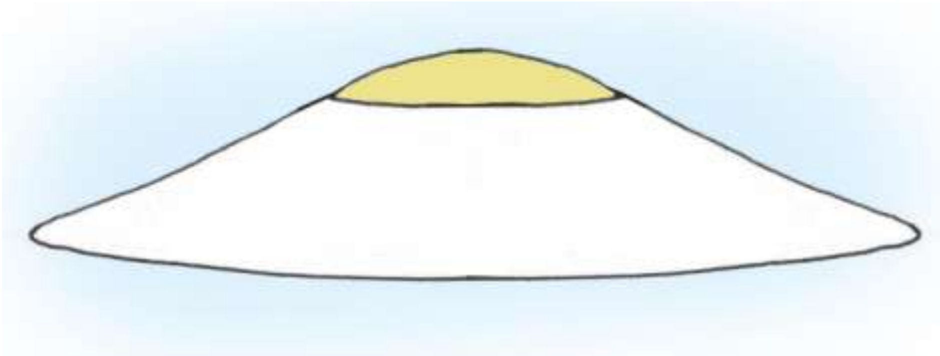
These two objects are in contact along a circle and along it the two surfaces have their common tangent plane.

You weld along this circle.

Then you remove the portion of the Flamm surface that you no longer need:



You remove the excess portion of the sphere:



And you get your result, by combining two portions of surfaces that represent portions of geometric solutions.

Now, what significance can these two mathematical solutions have, taken in isolation?

No ....

Taken separately, these two geometries do not describe the entire geometry in the region considered. Let us imagine a portion of space of constant curvature, which represents a star of constant density. How is the geometry outside. This object lacks its complement, the geometry it creates in the void that surrounds it.

It is the same for this geometry associated with this Flamm surface. Along this hypersurface one will find the quasi-Keplerian trajectories (which are not geodesics of the Flamm surface!). But then, where is the mass that creates this curvature of space?

For an object like the Sun, the curvatures are small. What is the equivalent of the passage connecting the two layers of the surface of Flamm, it is very tiny, being situated in the center of the star. As for the portion of sphere representing the interior of the Sun, this shield is almost flat.

We can however try to see what can happen when, keeping the density constant, we increase the radius of the star.

You know that in the heart of massive stars, a remnant in the form of an iron sphere is formed. When the massive star evolves into a supernova it collapses on this iron core and, by compressing it, transforms it into a neutron star, as conjectured by Fritz Zwicky in the 1930s.

The stars turn on themselves. When an ice skater brings her arms to her side, her speed of rotation increases. In the same way, at the end of this process the neutron stars turn like spinning tops. Usually up to 1000 revolutions per second. The neutrons are then pressed against each other.

By the way, what causes protons and electrons to disappear?

In nature the isolated neutron is unstable. It decomposes spontaneously after 15 minutes into proton and electron[16].

In the universe the particles, to "exist" must be able to fit their "wave function". Because particles are also waves. The order of magnitude of the associated wavelength is given by the Compton length :

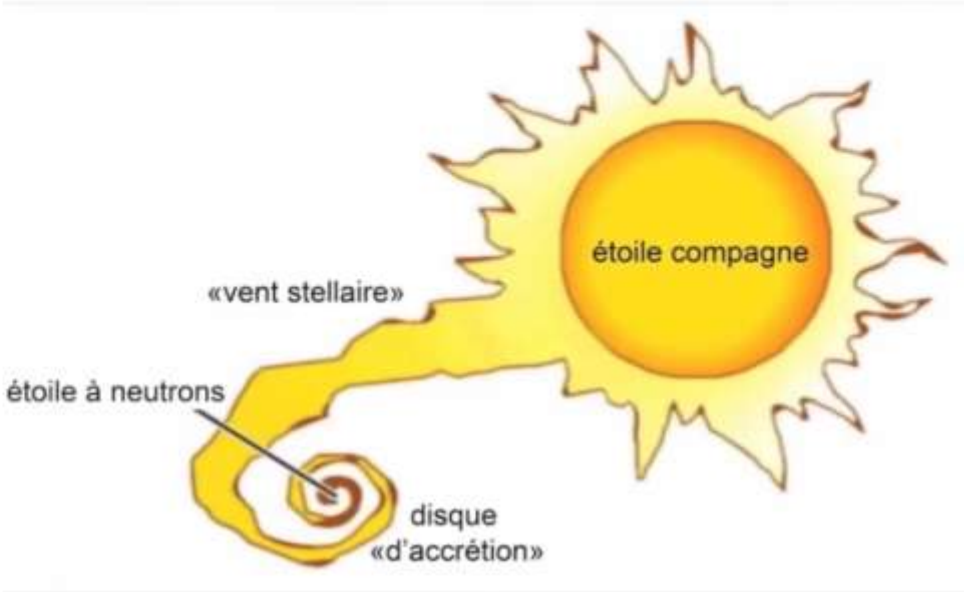
$$\lambda_c = \frac{h}{mc}$$

We see immediately that it varies as the inverse of the mass. Thus, the lighter the particles are, the more space they occupy in space. There is a tendency in drawings that are supposed to represent the components of atoms to draw large protons and neutrons and tiny electrons. In fact the electron is 1850 times (the inverse of the ratio of masses) that the proton. A concept that I had illustrated in my comic book Big Bang :

[16] Plus one antineutrino.

So, to exist, you need space. Since electrons need 1850 times more space than neutrons, they will be the first to disappear, bringing with them the conversion of protons into neutrons.

When the process of formation of a neutron star is completed, these are practically in contact and one can assimilate this environment to fragile electron ampoules, squeezed against each other. One can intuitively understand that this resistance to pressure is not infinite. But, for the moment, let's just consider the very progressive increase of the mass of a neutron star, capturing the light effluents of hydrogen and helium sent by a companion star.



Neutron star capturing the matter emitted by a companion star.

This situation must be very frequent in galaxies where, first of all, half of the stars form double systems. In many of these systems the stars of these double systems are made of massive stars and are close to each other.

The Sun emits a "solar wind". The same is true for massive stars, which can thus emit a quantity of matter equivalent to several solar masses. Thus, this scheme where one of the two massive stars has already transformed into a neutron star and collects matter emitted by its companion, before the latter also transforms into a neutron star is perfectly realistic. This flux remains very modest and it will take hundreds of millions of years for the gain in mass to bring the neutron star into a state of criticality that we will now consider, from its geometric angle.

Three of the previous drawings evoked the geometrical structure of a neutron star. All these stars have a common parameter: their density ρ_o which can be considered as a constant. Thus the Schwarzschild model is relevant. The geometric structure of the object thus results from the conjugation of a portion of Flamm's surface and a sphere of constant radius, since its value derives only from the value of ρ_o.

$$\boxed{\widehat{R} = \sqrt{\frac{3\,c^2}{8\pi G\rho_0}}}$$

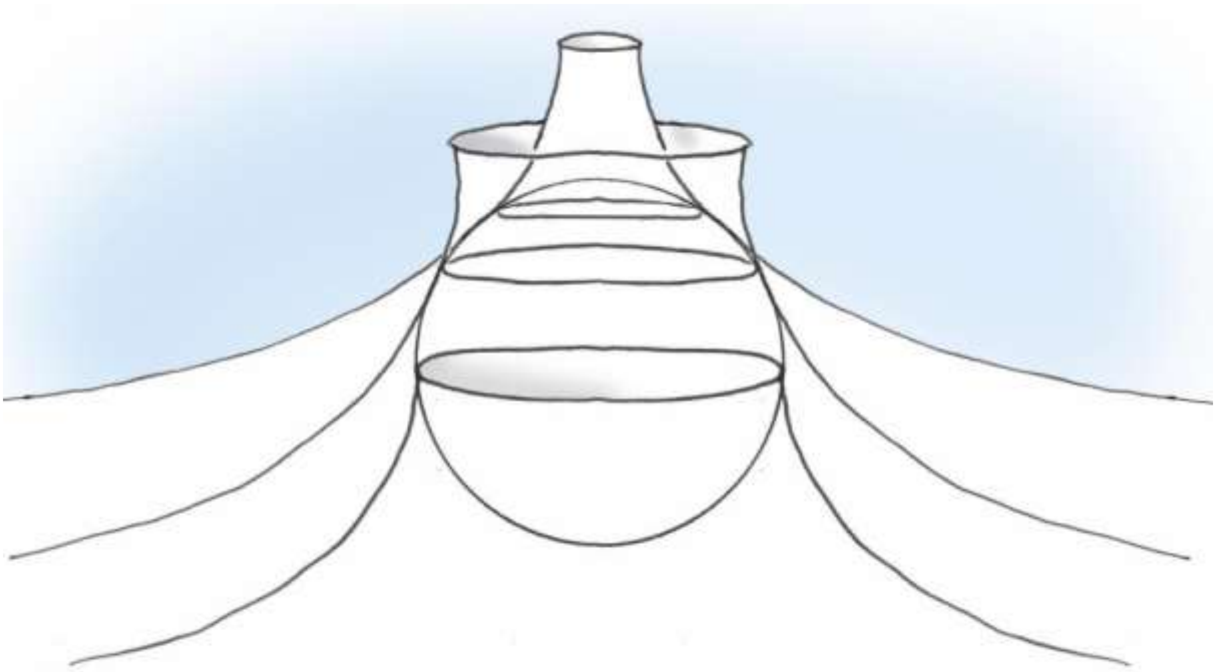
What will evolve according to the radius of the star is the surface of Flamm, whose throat circle has perimeter $2\,\pi R_s$ où R_s is the Schwarzchild radius, which grows as the mass, thus as the cube of the radius of the star.

$$\boxed{R_s = \frac{2GM}{c^2} = \frac{2G}{c^2}\frac{4}{3}\pi R^3 \rho_0 = \frac{8\pi G \rho_0}{3\,c^2} R^3 = \frac{R^3}{\hat{R}^2}}$$

The drawing below illustrates what will happen. As the radius of the star increases the radius of the Schwarzschild, which is deduced, will catch up with the radius " r chapeau ».

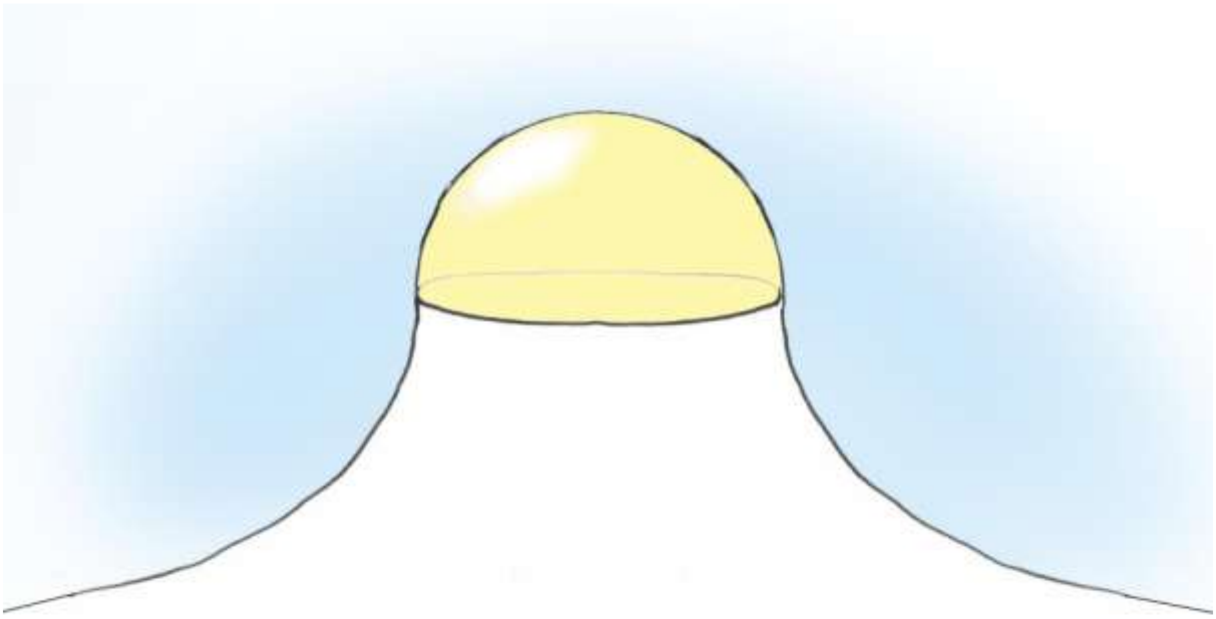
The Schwarzschild radius grows faster than that of the star.

Here is the evolution of the two surfaces together:



Towards a geometric criticality.

We end up with a borderline situation:



Beyond that, the connection between the two surfaces is no longer possible. The mass of the corresponding neutron star corresponds to a mass density in a sphere of radius r hat, which corresponds to 3 solar masses.

But, in this scenario of a rise to criticality there is a point that has totally escaped cosmologists. But Schwarzschild, as a fine physicist, immediately noted it in his 1916 article.

He is then the first to have envisaged (in 1916!) a possible variation of the speed of light. Here is the passage in question, first in German:

4. Die Lichtgeschwindigkeit in unserer Kugel wird:

$$v = \frac{2}{3\cos\chi_a - \cos\chi}, \qquad (44)$$

sie wächst also vom Betrag $\frac{1}{\cos\chi_a}$ an der Oberfläche bis zum Betrag $\frac{2}{3\cos\chi_a - 1}$ im Mittelpunkt. Die Druckgröße $\rho_0 + p$ wächst nach (10) und (30) proportional der Lichtgeschwindigkeit.

Im Kugelmittelpunkt ($\chi = 0$) werden Lichtgeschwindigkeit und Druck unendlich, sobald $\cos\chi_a = 1/3$, die Fallgeschwindigkeit gleich $\sqrt{8/9}$ der (natürlich gemessenen) Lichtgeschwindigkeit geworden ist. Es

Here is the translation in French:

4. **La vitesse de la lumière dans notre sphère est:**

$$v = \frac{2}{3\cos(\chi_a) - \cos(\chi)} \qquad (44)$$

de sorte qu'elle varie à partir de la valeur sur la surface

$$\frac{1}{\cos(\chi_a)}$$

jusqu'à la valeur au centre

$$\frac{2}{3\cos(\chi_a) - 1}$$

La variable de pression $\rho_0 + p$ augmente selon (10) et (30) proportionnellement à la vitesse de la lumière.

Au centre de la sphère $(\chi = 0)$, **la vitesse de la lumière et la pression deviennent infinies (...)**

dès que $\cos(\chi_a) = \frac{1}{3}$, la vitesse de chute est devenue égale à $\sqrt{\frac{8}{9}}$ de la vitesse de la lumière (mesurée naturellement).

In his article Schwarzschild gives the value 1 to the speed of light in vacuum. By performing the change of variable from its coordinate to the coordinate to the coordinate r on obtient :

$$v = \frac{c}{\frac{3}{2}\sqrt{1 - \frac{R_a^2}{\hat{R}^2}} - \frac{1}{2}\sqrt{1 - \frac{R^2}{\hat{R}^2}}}$$

At the center of the star this speed is :

$$v_o = \frac{c}{\frac{3}{2}\sqrt{1 - \frac{R_a^2}{\hat{R}^2}} - \frac{1}{2}}$$

The denominator of this fraction cancels for

This means that when the radius of the neutron star reaches this value, the speed of light becomes infinite at its center. We then obtain a second, lower value of its critical mass:

→Mc = 2.5 solar masses

This value corresponds to a critical angular value:

$$\chi_{ac} = \arccos(1/3)$$

That is 70.55°.

Still in his paper of February 1916 Karl Schawrzchild gives the way the pressure inside the star varies :

so verwandeln sich die Gleichungen (13), (26), (10), (24), (25) durch elementare Rechnung in die folgenden:

$$f_2 = \frac{3}{\varkappa\rho_0}\sin^2\chi, \quad f_4 = \left(\frac{3\cos\chi_a - \cos\chi}{2}\right)^2, \quad f_1 f_2^2 f_4 = 1. \qquad (29)$$

$$\rho_0 + p = \rho_0 \frac{2\cos\chi_a}{3\cos\chi_a - \cos\chi} \qquad (30)$$

$$3x = r^3 = \left(\frac{\varkappa\rho_0}{3}\right)^{-3/2}\left[\frac{9}{4}\cos\chi_a\left(\chi - \frac{1}{2}\sin 2\chi\right) - \frac{1}{2}\sin^3\chi\right]. \qquad (31)$$

Die Konstante χ_a bestimmt sich aus Dichte ρ_0 und Radius r_a der Kugel nach der Relation:

By making reappear the value of the speed of light in vacuum, taken equal to 1 by Schwarzschild :

$$p = \rho_0 c_0^2 \frac{\cos\chi - \cos\chi_a}{3\cos\chi_a - \cos\chi}$$

A pressure that is zero at the surface of the star and that, like the speed of light becomes infinite at the center when the radius of the star reaches the same critical value. This corresponds to the same value of the critical radius, shown above.

It should be remembered that pressure, in addition to being a measure of force per unit area, is also a volume density of energy.

We can see that it is impossible to conceive a set inner solution - outer solution of the Einstein equation able to describe the geometry inside and outside of a neutron star whose mass exceeds 2.5 solar masses, otherwise the pressure and the speed of light become infinite in its center.

It would therefore be appropriate to examine this specific point.

For more than fifty years cosmologists and astrophysicists have made the most complete impasse on this question. But this is not the first time this has happened. We have seen, with this question of primordial antimatter, that when theorists do not see any possibility to deal with a problem, they simply put it under the carpet.