Chapter 3 :

Calculate, there is nothing to see.

In 1905 Albert Einstein, then aged 26, showed that light does not propagate in an ether. Although it behaves like a wave, it corresponds to something much deeper, to the geometric structure of the universe.

Einstein's highly original personality lies in the fact that, while striving to integrate the most advanced mathematics, he remained a fantastic physicist, totally anchored in experimental and observational facts. It is thus significant that the Nobel Prize was awarded to him in 1921, not for relativity, special or general, but for his explanation of the photoelectric effect.

The conceptual leap represented by the introduction of his special relativity is based on the idea that time is only the fourth coordinate of a four-dimensional space, which he refers to as space-time. Under these conditions this fourth dimension must be measured .... In metres, or ... lengths in seconds.

Before that, time seemed to be of a different nature. Space, three-dimensional, was alien to time. The focus was on the Pythagorean theorem:

- The square of the hypotenuse is equal to the sum of the squares of the other two sides.

Einstein's discovery can be expressed in a totally geometrical way by saying:

- We live in a four dimensional Minkowski space where the square of the hypotenuse is equal to the difference of the squares of the other two sides.

The mathematician-geometer Hermann Minkowski

Explosion of absurdity! This space therefore contains right-angled triangles which, while having the two sides adjacent to this right angle non-zero, have their third side reduced to zero.

One is then tempted to ask what mental image these new 'relativistic' physicists and mathematicians-geometers have of this new representation of the universe. The answer is that they do not. They only 'see' the object of their study through the equations they draw on the pages of their articles. An image? They don't think they need it.

When their interlocutor cannot, in turn, look at these pages covered with hieroglyphs, which only mathematicians can understand, they do not feel obliged to provide others with what they themselves do not possess: an image. One could compare their reaction to that of the Mary Poppins character.

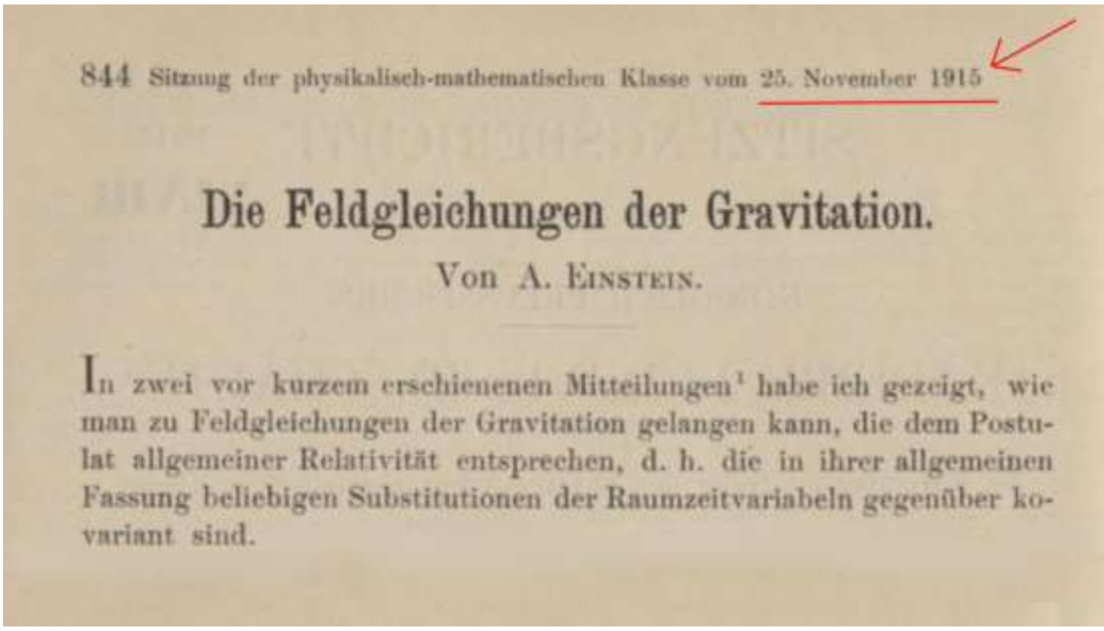
Mary Poppins: "I never explain anything

The century that began in 1900 marked the end of all pictorial representation. This could be summed up by the phrase, paraphrasing those of police officers dealing with a crowd:

- Calculate, there's nothing to see!

With his special relativity, Einstein provided the answer to the problem posed by Michelson and Morley's experiment.

In 1915 he extended his model by laying the foundations of general relativity. Here is his famous article:



844 Sitzung der physikalisch-mathematischen Klasse vom 25. November 1915

Die Feldgleichungen der Gravitation.

Von A. Einstein.

In zwei vor kurzem erschienenen Mitteilungen[1] habe ich gezeigt, wie man zu Feldgleichungen der Gravitation gelangen kann, die dem Postulat allgemeiner Relativität entsprechen, d. h. die in ihrer allgemeinen Fassung beliebigen Substitutionen der Raumzeitvariabeln gegenüber kovariant sind.

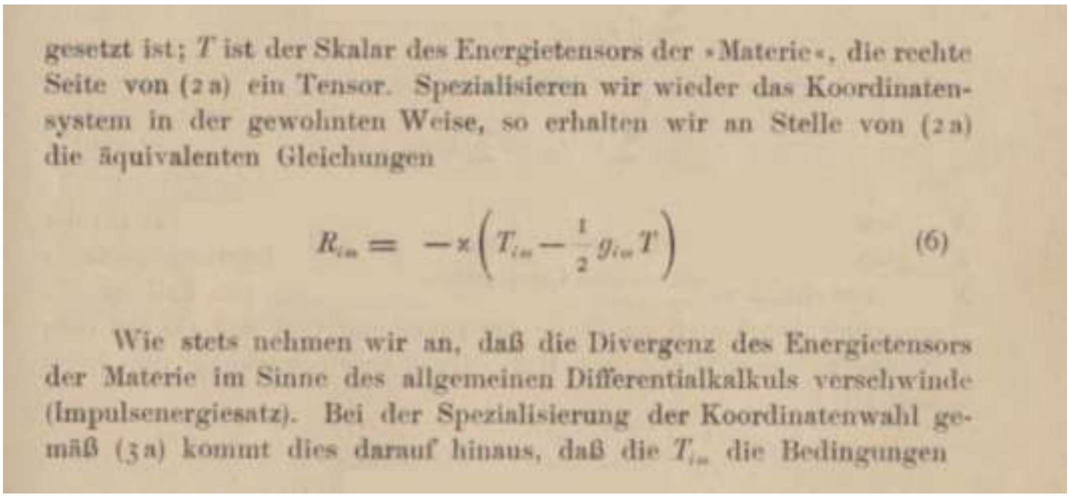
Einstein's article of November 1915, laying the foundations of his general relativity
Title : Field equation of General Relativity.

Minkowski's space was highly disconcerting, but it was still flat, free of curvature. In accordance with the central idea of general relativity:

Energy-matter content curvature

This Minkowski space can only describe the geometry of empty spaces. General relativity is more ambitious, undertaking to manage all regions of the universe, from the most rarefied to the densest.

It can be summed up entirely in the equation published by Einstein in 1915, in his paper : . Here it is, taken from his 1915 article:



gesetzt ist; T ist der Skalar des Energietensors der »Materie«, die rechte Seite von (2a) ein Tensor. Spezialisieren wir wieder das Koordinatensystem in der gewohnten Weise, so erhalten wir an Stelle von (2a) die äquivalenten Gleichungen

$$R_{im} = -\varkappa\left(T_{im} - \frac{1}{2}g_{im}T\right) \qquad (6)$$

Wie stets nehmen wir an, daß die Divergenz des Energietensors der Materie im Sinne des allgemeinen Differentialkalkuls verschwinde (Impulsenergiesatz). Bei der Spezialisierung der Koordinatenwahl gemäß (3a) kommt dies darauf hinaus, daß die T_{im} die Bedingungen

The original formulation of the Einstein field equation.

We will not explain here what the terms of this equation mean. But we will provide elements to illustrate a rather tasty anecdote. . At the time when Einstein was trying to reinterpret the world on new bases, a great mathematician, David Hilbert, was already internationally renowned for his rich and varied work.

It is known that Einstein was never very comfortable with mathematics. His genius is based above all on his very thorough understanding of the phenomena of physics and his ability to question the knowledge of his time. One could say that he was a pioneer in applying the mathematics of his time to physics. But this meeting was also linked to that of the Swiss mathematician Marcel Grossman, who was the same age as him, when Einstein was granted a position in Zurich.

Marcel Grossman in 1909

Grossman founded the Swiss Mathematical Society and became head of the Swiss Federal Institute of Technology in Zurich. For many years he followed the work and ideas of Einstein. The model of general relativity can only emerge if we use the mathematical tool of tensors. It was Grossman who introduced Einstein to this aspect of mathematics. They will co-author several documents, brochures and articles.

It is not wrong, at least for general relativity, to say that it was born of the collaboration between Grossman and Einstein, the former helping the latter to formalise his ideas mathematically.

At that time, the exchange between Einstein and Grossman was the exception, the worlds of physics and mathematics being practically disjointed. In fact, and this is true throughout the chaotic history of science, mathematics is always decades ahead of physics. This implicitly means that, conceptually, mathematics is progressing faster.

There is a reason for this. Mathematics enjoys greater freedom and is not required to fit into a physical or observational reality. Mathematicians do not care whether their equations will be related to any physical reality in the future. The laboratory of a mathematician is his paper and pencil.

An example of maths ahead of physical reality is the work of the German mathematician Bernhard Riemann, a pioneer in the world of curved spaces with more than two dimensions, to which he left his name. We said earlier that special relativity boiled down to the idea that the geometric description of the universe could only be done by using a four-dimensional space, a space-time, which happened to be a Minkowski space.

Bernhard Riemann 1826 - 1866

The transition to general relativity is reflected in the extension of the geometrical context from Minkowski space to a four-dimensional Riemann space. The Minkowski space being a special case of "flat" Riemann space, an example of curvature. Just as a plane is a special case of a curved surface, ... without curvature.

Riemann's contribution is thus one of the keys to the deployment of this new vision of the universe. He developed this in his thesis in 1854 without imagining for a second that his work could be used in astrophysics (in this case to explain the deviation of Mercury's perihelion due to the curvature of space).

A great Russian mathematician, Mikhail Ostrogradski, of international stature, commented on this work by saying:

- It is hard to see what purpose these three-dimensional curved space geometries could serve, since it is obvious that the space we live in is Euclidean.

Thanks to Grossman, Einstein found the use of the mathematical tools of his time.

From the outset, the mathematical advances appear much more sophisticated than those used by engineers, such as the Maxwell and Navier-Stockes equations presented above. The mathematician David Hilbert was a great producer of this advanced mathematics, not knowing that his inventions would find a key place in what would become mathematical physics. Indeed, quantum mechanics is based entirely on so-called Hilbert spaces, which are eminently abstract.

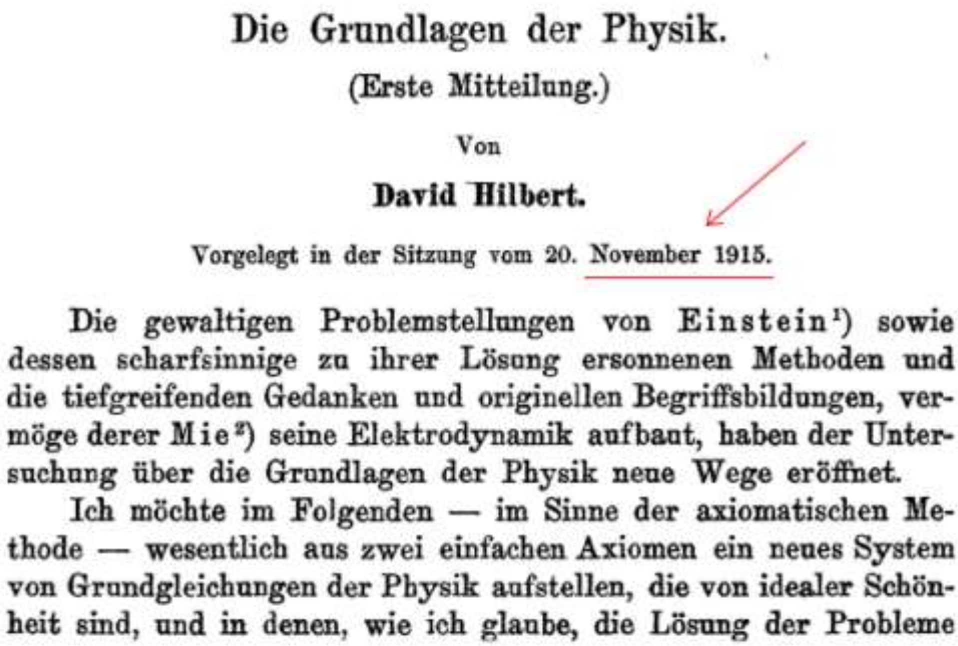
The mathematician David Hilbert was 53 years old when
Einstein published his general relativity.

When Einstein published his special relativity, David Hilbert was convinced that he was living in a universe that had no relation to reality, the world of physics. Here is another, rather tasty anecdote. At that time one of his colleagues gave an annual lecture at an engineering school, talking about some advances in mathematics. That year this colleague was ill and Hilbert was asked to replace him. When he takes his seat in front of these young engineers, he says straight away:

- It is often said that mathematicians and engineers have difficulty understanding each other. This is not true. They simply do not belong together.

In Göttingen Einstein met Hilbert and spent many hours trying to get him to change his position. In successive meetings over the course of a year, Hilbert realised that these mathematical tools would enable him to create a comprehensive theory of physical phenomena. Let's remember that the physics of that time can be reduced to two types of phenomena, to two worlds. There is electromagnetism on the one hand, and gravitation on the other. Two other forces, called the strong interaction and the weak interaction, would only appear later. Thus, at that time, whoever succeeds in creating a mathematical context encompassing these two forces creates at the same time what today would be called a 'theory of everything'. The mathematics he used were, on the one hand, differential geometry, which allowed the manipulation of Riemann's curved spaces, and on the other hand, the calculus of variations, which had been developed a century earlier in the forges of various mathematicians, including the Frenchman Joseph-Louis Lagrange (1736-1813).

While Albert Einstein, the hard-working tortoise, was making progress, Hilbert, the thoroughbred, took his race in stride and in a few months wrote a long article entitled "Foundations of Physics":



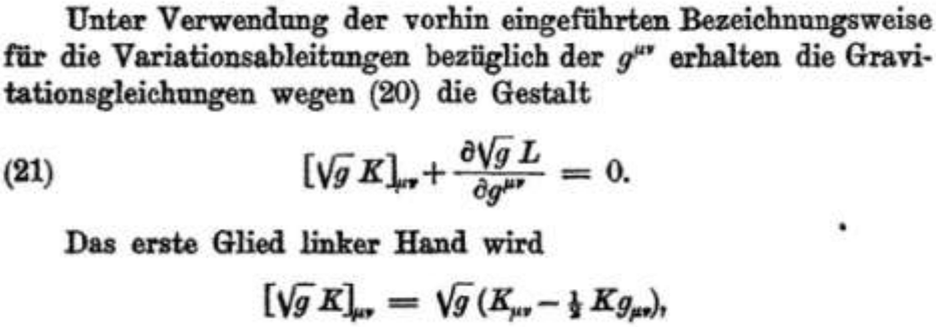
Die Grundlagen der Physik.
(Erste Mitteilung.)
Von
David Hilbert.
Vorgelegt in der Sitzung vom 20. November 1915.

Die gewaltigen Problemstellungen von Einstein[1]) sowie dessen scharfsinnige zu ihrer Lösung ersonnenen Methoden und die tiefgreifenden Gedanken und originellen Begriffsbildungen, vermöge derer Mie[2]) seine Elektrodynamik aufbaut, haben der Untersuchung über die Grundlagen der Physik neue Wege eröffnet.

Ich möchte im Folgenden — im Sinne der axiomatischen Methode — wesentlich aus zwei einfachen Axiomen ein neues System von Grundgleichungen der Physik aufstellen, die von idealer Schönheit sind, und in denen, wie ich glaube, die Lösung der Probleme

David Hilbert's article of 20 November 1915.

Note the date of submission of the manuscript, 20 November 1915, and refer to Einstein's. What do you notice? Einstein's article was submitted five days later. But what does Hilbert's article contain? . This equation :



Unter Verwendung der vorhin eingeführten Bezeichnungsweise für die Variationsableitungen bezüglich der $g^{\mu\nu}$ erhalten die Gravitationsgleichungen wegen (20) die Gestalt

$$[\sqrt{g}\,K]_{\mu\nu} + \frac{\partial\sqrt{g}\,L}{\partial g^{\mu\nu}} = 0. \tag{21}$$

Das erste Glied linker Hand wird

$$[\sqrt{g}\,K]_{\mu\nu} = \sqrt{g}\,(K_{\mu\nu} - \tfrac{1}{2}\,K g_{\mu\nu}),$$

It concerns a mathematical object designated by the letter K. This tensor is designated by Einstein with the letter R[1]. Combining these two lines of equations gives :

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[1] Flanked by two indices it is the "Ricci tensor". Without these indices it is the "Ricci scalar".

$$\sqrt{g}\,(R_{\mu\nu}-\frac{1}{2}Rg_{\mu\nu})=-\frac{\partial\sqrt{g}\,L}{\partial g^{\mu\nu}}$$

The second member, and this corresponds to elements of differential geometry, can be identified with a tensor multiplied by root of g. We will then have :

$$\sqrt{g}\,(R_{\mu\nu}-\frac{1}{2}Rg_{\mu\nu})=-\sqrt{g}\,T_{\mu\nu}$$

This term is then a factor on both the first and second member. It can be removed. Il vient :

$$R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R=-\;T_{\mu\nu}$$

This corresponds to the way Einstein's equation is formulated today, modulo a constant factor in the second, negative member, "the Einstein constant". At this stage, this treatment would be sufficient to conclude "that it is indeed the Einstein field equation", minus the cosmological constant that will be added later. However, the properties of these tensors mean that this equation is equivalent to :

$$R_{\mu\nu}=-(T_{\mu\nu}-\frac{1}{2}g_{\mu\nu}T\;)$$

That's it. All that remains is to add the Einstein constant, which Hilbert takes here to be equal to unity, and we get :

$$R_{\mu\nu}=-\;\chi(T_{\mu\nu}-\frac{1}{2}g_{\mu\nu}T\;)$$

If you replace these indices m and n by the letters i and m, check, it is exactly the equation (6) published by Einstein according to a deposit made ... five days later!

I hope the reader will not be put off by the fact that I have put some equations, admittedly very obscure ones, in this text. Let us say that it is to shed some light on a point in the history of science, which has probably never been described with such precision.

Einstein is devastated. He knows that this equation is a very important result. Incidentally, he has just published his first solution of the equation without a second member, when the tensor T is zero, describing the field in the vacuum surrounding a mass, around a star, and this calculation explains the advance of Mercury's perihelion. So it's a major conceptual revolution. And he's just had the prize for his work stolen, give or take a few days, by a man to whom he's given all his secrets, all his ideas.

He was unaware that Hilbert was running at high speed towards this result, which he himself had reached at the cost of a more artisanal than mathematical approach. The following days and weeks are quite turbulent. There are letters between these two prominent figures. Finally, the tension disappears. Einstein wrote to Hilbert that the most important thing for him was that their friendship should not be affected. And all is well.

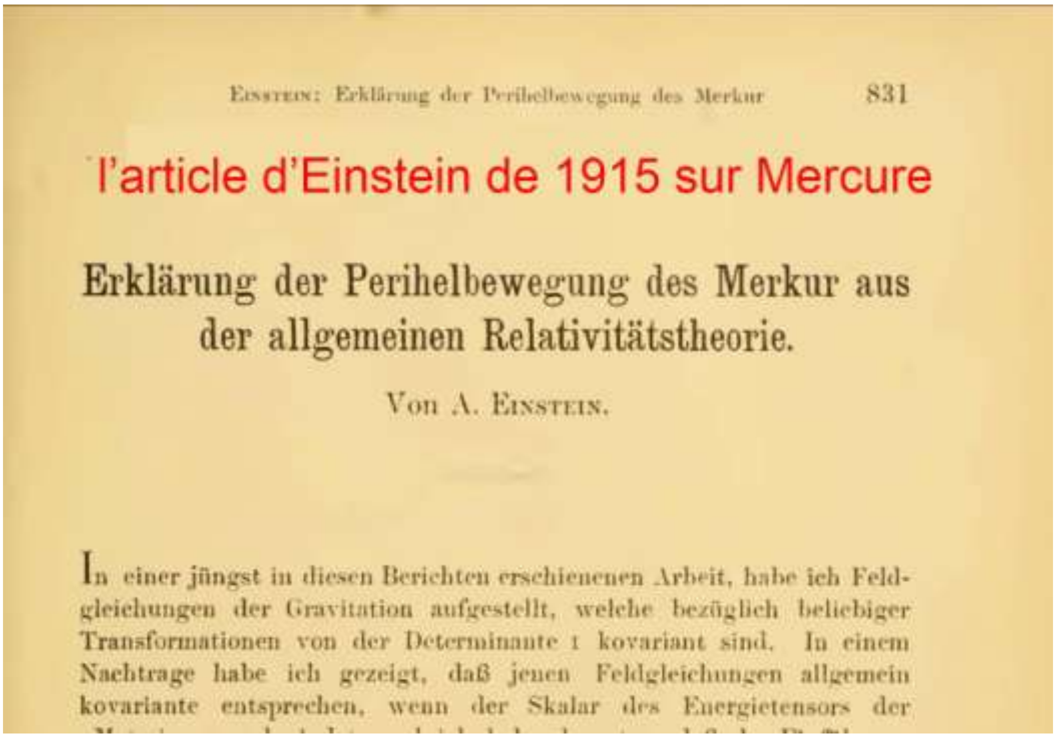
Hilbert does not need to attach his name to this discovery. His track record is already very good. This field equation will henceforth be known as the Einstein equation, making it well known in all countries of the world.

In 1914 the First World War broke out. Many talented young scientists will lose their lives. This was the case for the young Werner Boy, a pupil of Hilbert's, to whom we shall return later, who was killed in a trench in France one month after the beginning of the conflict.

Karl Schwarzschild is a mathematician and astronomer. At that time it was common for men to perform both functions. At the time the conflict broke out he was 42 years old and the father of two young children. But he immediately enlisted as an officer out of patriotism and was posted to the Eastern Front.

Karl Schwarzschild.

He followed with passion the emergence of special relativity. Receiving the journal in which Einstein published his many papers, he saw, over time, this fantastic field equation emerge. He also read the article, published at the same time, in which he presents his approximate solution, which accounts for the drift of the perihelion of the planet Mercury.



Einstein: Erklärung der Perihelbewegung des Merkur 831

l'article d'Einstein de 1915 sur Mercure

Erklärung der Perihelbewegung des Merkur aus der allgemeinen Relativitätstheorie.

Von A. Einstein.

In einer jüngst in diesen Berichten erschienenen Arbeit, habe ich Feldgleichungen der Gravitation aufgestellt, welche bezüglich beliebiger Transformationen von der Determinante 1 kovariant sind. In einem Nachtrage habe ich gezeigt, daß jenen Feldgleichungen allgemein kovariante entsprechen, wenn der Skalar des Energietensors der

Albert Einstein's paper presenting the first approximate solution
of his field equation without second member.

In January and February 1916 he published two papers, one after the other. These were the first exact solutions (free of approximations) of the Einstein equation in a stationary situation (time independent solutions). In the first he produces the equation that describes the geometry in the vacuum that surrounds a mass, a star. When he applied it to the case of the Sun he found the approximate solution published a few months earlier by Einstein, which was more than sufficient, given the small deviation from Newtonian mechanics.

SCHWARZSCHILD: Über das Gravitationsfeld eines Massenpunktes 189

Sitzber. Deut. Akad. Wiss. (1916)

Über das Gravitationsfeld eines Massenpunktes nach der EINSTEINschen Theorie.

Von K. SCHWARZSCHILD.

(Vorgelegt am 13. Januar 1916 [s. oben S. 42].)

§ 1. Hr. EINSTEIN hat in seiner Arbeit über die Perihelbewegung des Merkur (s. Sitzungsberichte vom 18. November 1915) folgendes Problem gestellt:

Ein Punkt bewege sich gemäß der Forderung

$$\left.\begin{aligned} &\delta\int ds = 0, \\ \text{wobei} \quad & ds = \sqrt{\sum g_{\mu\nu}dx_\mu dx_\nu} \quad \mu,\nu = 1,2,3,4 \end{aligned}\right\} \quad (1)$$

The first article by Karl Schwarzschild.

Translation: gravitational field created by a point mass according to Einstein's theory. The first translation of this paper, in English, will only be available in 1975.

A month later he constructed the geometry inside a sphere filled with an incompressible fluid (a stole where the density is considered constant). To construct this geometry, i.e. to produce a solution in the form of a "metric" which makes it possible to calculate all the possible trajectories of "control masses" outside and inside the star.

To imagine that objects can pass through the mass of a star seems a far cry from the world of physics. In the Schwarzschild era, yes. But later we will see the appearance of particles which, interacting very weakly with matter, are capable of crossing large masses from one side to the other. So this solution is not totally unrealistic. What is important is that Schwarzschild manages the connection of these solutions perfectly, on the surface of the star.

But, having contracted an infection on the front, he died a few months later. The scientific community therefore only remembers Einstein's name and the fact that he solved the paradox raised by Mercury's trajectory. This dispels two of the three clouds identified by Lors Kelvin, the first of which was the question of the constancy of the speed of light, elucidated by the special relativity introduced by Einstein in 1905, with this relation establishing an equivalence between assembly and energy:

E = mc2

The most spectacular irruption of a new physics is obviously the implementation of nuclear energy in the form of A and H bombs. It is no exaggeration to say that this exception is comparable to the invention of fire.

The second outdoor experiment confirming the new physics.

It is an implementation that involves a profound paradigm shift. Indeed, if fusion reactions can occur it is because neutrons can penetrate the nuclei of uranium atoms. This is only possible because of the tunnel effect.

Indeed, quantum mechanics gives up locating objects in a deterministic way. It simply assigns them "probabilities of presence". If neutrons manage to penetrate nuclei it is because when they brush against them there is a non-zero probability that they are already inside.

If this tunneling effect is obvious to a specialist in quantum mechanics, it marks the complete bankruptcy of common sense in that physics. As the French derive a large part of the electrical energy they consume, they exploit this magnificently absurd tunnel effect on a daily basis.

If the man in the street benefits daily, without being aware of it, from the exploitation of this physics woven with paradoxes, he might think that the mode of relativity is only of interest to specialists, in their laboratories, or to astrophysicists and cosmologists, who concentrate on distances that no longer have anything to do with the scales of the earth itself or the solar system.

Is that not correct? General relativity predicts that clocks do not give the same measurements away from any mass, or near one. Thus, time does not flow at the same rate at the Earth's surface and at a certain height. Two American researchers, Robert Pound and Glenn Rebka, highlighted this small difference in 1960, the difference in altitude being 22.5

metres. I will not venture to describe the principle of the measurement [2]. The fact remains that the effect could be demonstrated, first with an accuracy of 10% and then 1%.

But again, these are laboratory experiments. There are situations where these small errors add up over time to large errors. This is the case with the GPS system. It is based on a set of satellites orbiting the earth, each containing an atomic clock. For geolocation using this satellite system to work, it is essential that all these clocks give the time in a meaningful way. It is therefore imperative to take into account the temporal drift, compared to a time measurement at zero altitude The instantaneous error is small. But if we reason on the scale of days, months, years, if we do not take this correction into account, the GPS system would give cumulative errors on the positions, such that it would become unusable.

Thus, when you follow the directions of the GPS of your vehicle, you exploit the achievements of Einstein's model, of general relativity.

Just as the nineteenth century represented the benefit of progress in physics and chemistry, so our era is also living on the achievements of the second age of science, corresponding to a period from 1986 to the turn of the seventies. Since the turn of the seventies we have done nothing but develop technological applications.

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[2] Using Mossbauer spectroscopy.